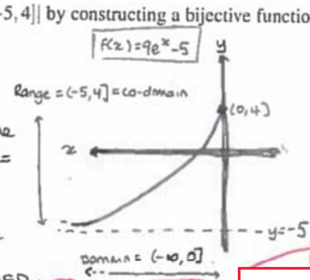
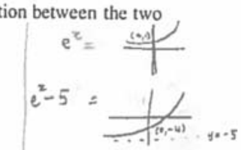
QUESTION 5. (3 points) Convince me that $|(-\infty, 0]| = |(-5, 4]|$ by constructing a bijective function between the two sets.

Let $f : (-\infty, 0] \longrightarrow (-5, 4]$

$\therefore f(x) = 9e^x - 5$.





$me^0 - 5 = 4$

$m - 5 = 4 \Rightarrow m = 9$.

From graph, it is clear that f(x) is one-to-one (since function is increasing) and onto (since range = co-domain).

Hence f is a bijective function.

le know from fact that, if a bijective function can be uilt, the domain & codomain have same cardinality.

$\therefore |(-\infty, 0]| = |(-5, 4]|$. QED

QUESTION 6. (6 points)

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Fact: If

$n = 35$

$\phi(n) =$

gcd(8, 3

$\therefore 8^{24}$ (

Fact: There is no bijective function f from $\mathbb{R} \to \mathbb{N}$
Hence $\mathbb{R}$ is uncountable and $|\mathbb{R}| \neq |\mathbb{N}|$

Question: Convince me that $|(-\infty, 0)| = |(-3, 1)| = |\mathbb{R}|$
We show $|(-\infty, 0)| = |(-3, 1)|$

Produce a function: $f: (-\infty, 0) \longrightarrow (-3, 1)$
$f(x) = 4e^x - 3$

Hence $|(-\infty, 0)| = |(-3, 1)|$

Side note: suppose asked to show $|(-\infty, 0]| = |(-3, 1)|$
Since $|(-\infty, 0)| = |(-3, 1)|$
and $(-\infty, 0) \cup \{0\} = (-\infty, 0]$
We know $|(-\infty, 0]| = |(-\infty, 0)| = |(-3, 1)|$

Homework 10

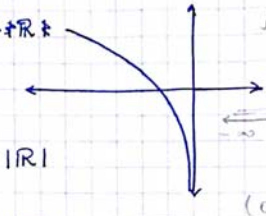
I) i. Convince me that $|\mathbb{R}| = |(-4, 1]|$

<u>ans</u>: <u>first</u> show ~~0, ∞~~ $|(-\infty, 0]| = |\mathbb{R}|$

$f: (-\infty, 0) \to \mathbb{R}$
$f(x) = \ln(-x)$

is bijective

$\therefore |(-\infty, 0)| = |\mathbb{R}|$

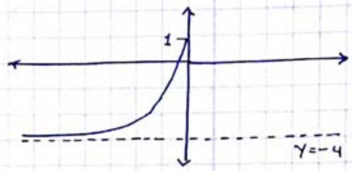


$(-\infty, 0) \cup \{0\} = (-\infty, 0]$
$|(-\infty, 0]| = |(-\infty, 0)| = |\mathbb{R}|$
So $|(-\infty, 0]| = |\mathbb{R}|$

second: show $|(-\infty, 0]| = |(-4, 1]|$

$g: (-\infty, 0] \to (-4, 1]$
$g(x) = 5e^x - 4$

is bijective



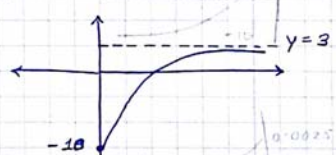
$\therefore |(-\infty, 0]| = |(-4, 1]|$ and $|(-\infty, 0]| = |\mathbb{R}|$
Hence $|\mathbb{R}| = |(-4, 1]|$

Since $\mathbb{R}$ is uncountable, $(-4, 1]$ is also uncountable

ii. Convince me that $|(-10, 3)| =$ ~~0, ∞~~ $|(0, 0.0025)|$.
<u>ans</u>: first: show $|(0, \infty)| = |(-10, 3)|$

$f: (0, \infty) \to (-10, 3)$
$f(x) = -13e^{-x} + 3$

is bijective



$\therefore |(0, \infty)| = |(-10, 3)|$

Second: show $|(0, \infty)| = |(0, 0.0025)|$

$g: (0, \infty) \to (0, 0.0025)$
$g(x) = -0.0025e^{-x} + 0.0025$

is bijective

$\therefore |(0, \infty)| = |(0, 0.0025)|$ and $|(0, \infty)| = |(-10, 3)|$
Hence $|(-10, 3)| = |(0, 0.0025)|$

iii) Is the set $A = (0, 0.000033) \cap \mathbb{Q}$ finite or infinite. Is A countable?
<u>ans.</u> Betw. any two rational numbers there are infinitely many rational numbers so A is infinite. $A \subset \mathbb{Q}$. Since $\mathbb{Q}$ is countable, so is A.

<u>Rule</u>. Subset of countable set $\to$ countable eg $(-\frac{1}{2}, \frac{1}{2}) \subset \mathbb{Q} \to$ countable
subset of uncountable set $\to$ countable or uncountable.
eg. $\mathbb{Q} \subset \mathbb{R} \to$ countable, $\mathbb{Z} \subset \mathbb{R} \to$ countable, $(0, 1) \subset \mathbb{R} \to$ uncountable

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